

Polarisation des umgebenden Lösungsmittels bewirkt wird¹³. Das bedeutet, daß im mikroskopischen Bereich eine höhere „effektive Dielektrizitätskonstante“ angenommen werden muß, als durch ϵ in Gl. (7) angegeben wird. Es hat sich daher als besser erwiesen^{2,3}, die Fluoreszenzmaxima von verschiedenen Hetero-Excimeren direkt zu vergleichen. Trägt man dementsprechend die Werte aus Abb. 4 gegen die $\tilde{\nu}_c$ -Werte des als Referenz gewählten Hetero-Excimeren Anthracen-Diäthylanilin (mit $\mu_c^2/\rho^3 = 4720 \text{ cm}^{-1}$) auf, so ergibt sich eine Darstellung mit wesentlich geringerer Streuung, da

spezifische Lösungsmittleigenschaften eliminiert sind.

Man erhält auf diese Weise für die Hetero-Excimeren:

3.4-Benzacridin-Diäthylanilin

$$\mu_c^2/\rho = 4100 \text{ cm}^{-1},$$

3.4-Benzacridin-p-Methoxydimethylanilin

$$\mu_c^2/\rho^3 = 4200 \text{ cm}^{-1}.$$

Mit einem Wert von $6 \pm 0.5 \text{ \AA}$ für den Äquivalentradius ρ ergibt sich daraus $\mu_c = 13,3 \pm 1,7$ Debye, ein Wert, wie er auch für vergleichbare Hetero-Excimere mit aromatischen Kohlenwasserstoffen gefunden wurde^{2,3,12}.

Herrn Professor Dr. A. WELLER möchte ich für anregende Diskussionen herzlich danken.

¹³ W. LIPTAY, H. WEISSENBACHER, F. TIEMANN, W. EBERLEIN u. G. KONOPKA, Z. Naturforsch. **23a**, 377 [1968].

Unipolar Flow of Charge Carriers in a Dense Gas with and without Consideration of Diffusion

W. DÄLLENBACH

Geneva Research Center of the Battelle Institute

(Z. Naturforsch. **25 a**, 1447—1452 [1970]; received 14 February 1970)

The basic equations for the unipolar, stationary, one-dimensional flow of charge carriers in a dense gas, characterized by mobility and diffusion coefficient, can be integrated numerically. The discharge gap generally has a finite length; as far as in the end cross-section either density of particles and intensity of electric field tend towards infinity, or the density of particles becomes zero. Which of these two cases occurs depends on the current density of the discharge and on the intensity of the electric field in the initial cross-section. The notions mobility and diffusion coefficient will lose their applicability close to a pole like singularity as well as in a "dilution to zero", so that from a certain cross-section onwards the continuation of discharge is determined by modified equations.

It is shown that in case the diffusion component of the current density is neglected, the integrals of the basic equations change fundamentally. Neglecting the diffusion is inadmissible. This is finally a consequence of the relationship between mobility and diffusion coefficient, as expressed by the Einstein-relation.

I. Description of the Problem

The transport of charge carriers in a dense plasma has been investigated already in the first quarter of this century in connexion with the treatment of processes in ionisation chambers¹.

Today, this problem awakens new interest, espe-

cially in connexion with electric discharges in the thermionic plasma of MHD-generators and the transport of electricity in semi-conductors.

To simplify the mathematical analysis, it was quite frequently proposed and tried to disregard the diffusion currents of charge carriers against the field current (ohmic part)². Formulas deduced by

Sonderdruckanforderungen an Dr. Ing. W. DÄLLENBACH, Sagerstrasse 6, CH 3006 Bern 16.

¹ H. STRÜCKLEN, Geiger-Scheel-Handbuch d. Physik XIV, 1–50 [1927]. — E. SCHWEIDLER, Wien-Harms-Handbuch d. Ex-

perimentalphysik XIII, Akademische Verlagsgesellschaft m.b.H., Leipzig, 1. Teil, 3–105 [1929].

² Concerning the neglect of diffusion against field currents, see STRÜCKLEN¹, p. 4–10, and SCHWEIDLER¹, p. 82–85.



Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland Lizenz.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung „Keine Bearbeitung“) beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermöglichen.

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

On 01.01.2015 it is planned to change the License Conditions (the removal of the Creative Commons License condition "no derivative works"). This is to allow reuse in the area of future scientific usage.

such simplification may be encountered as recently as in latest literature³.

The special problem of "unipolar, stationary, one-dimensional flow of charge carriers in a dense gas" can mathematically be solved rigorously. The significance of neglecting the diffusion currents against the field currents will be shown by means of this special case.

II. The Solutions with Considering the Diffusion

A plane electrode situated at $x=0$ emits charge carriers of one species to $x>0$ into a uniform dense gas at temperature T , e. g. positive or negative ions of the charge e , thus the latter may also be electrons. Provided there exists for $x>0$ an electric field in direction $x>0$, which depends only on x , there will result a unipolar plane flow of charge carriers whose temperature will also be T . In the stationary case, the current density j is independent of x and in particular

$$j = e n \mu E - e D (dn/dx) = \text{const.} \quad (1)$$

The mobility μ and the diffusion coefficient D will also be constants independent of x . The particle density n of the charge carriers is connected with the electric field E by the Poisson-equation.

$$\epsilon_0 (dE/dx) = e n. \quad (2)$$

ϵ_0 is the known constant of the system of units.

For given initial values

$$x=0; \quad n=n_0; \quad E=E_0; \quad (3)$$

n , E can be determined from (1), (2) as functions of x , n_0 , E_0 , e , μ , D and of the parameter j .

If the Einstein-relation

$$e D = \mu k T \quad (4)$$

is valid, where k means the Boltzmann constant, it is suitable to introduce the following new units:

$$\begin{aligned} \left(\frac{\epsilon_0 k T}{e^2 n_0} \right)^{1/2} & \quad (\text{cm}) & \text{Debye-length for } x, \\ n_0 & \quad (\text{cm}^{-3}) & \text{for } n, \\ \left(\frac{n_0 k T}{\epsilon_0} \right)^{1/2} & \quad (\text{V/cm}) & \text{for } E, \\ e n_0 \mu \left(\frac{n_0 k T}{\epsilon_0} \right)^{1/2} & \quad (\text{A/cm}^2) & \text{for } j. \end{aligned} \quad (5)$$

Equations (1), (2), (3) then reduce to

$$j = n E - dn/dx = \text{const}, \quad (6)$$

$$dE/dx = n, \quad (7)$$

$$x=0; \quad n=1; \quad E=E_0 \quad (8)$$

in the now dimensionless variables x , n , E , the parameter j , and the initial value E_0 .

n from (7) substituted into (6) and the latter integrated yields for E a Riccati differential equation; in particular because of (8)

$$\frac{dE}{dx} = \frac{1}{2} E^2 - j x + 1 - \frac{1}{2} E_0^2. \quad (9)$$

By substitutions

$$E = - \frac{d}{dx} (\ln \eta^2), \quad (10)$$

$$-j x + 1 - \frac{1}{2} E_0^2 = - (2 j^2)^{1/2} \xi. \quad (11)$$

(9) becomes

$$d^2 \eta / d\xi^2 = \xi \eta. \quad (12)$$

The general integral of this equation can be represented in a closed form by means of cylindrical functions⁵. For the numerical calculation by machine it is easier to use

$$\eta = C_1 S_1(\xi) + C_2 S_2(\xi) \quad (13)$$

where

$$\begin{aligned} S_1(\xi) &= 1 + \frac{1}{2 \cdot 3} \xi^3 + \frac{1}{2 \cdot 3 \cdot 5 \cdot 6} \xi^6 + \frac{1}{2 \cdot 3 \cdot 5 \cdot 6 \cdot 8 \cdot 9} \xi^9 + \dots, \\ S_2(\xi) &= \xi + \frac{1}{3 \cdot 4} \xi^4 + \frac{1}{3 \cdot 4 \cdot 6 \cdot 7} \xi^7 + \frac{1}{3 \cdot 4 \cdot 6 \cdot 7 \cdot 9 \cdot 10} \xi^{10} + \dots \end{aligned} \quad (14)$$

The constants of integration C_1 , C_2 may be determined, except for an insignificant factor due to

(10), by inserting the initial values (8) in (10), (11), (13), (14).

³ E. RUTHERFORD, Phil. Mag. (6) **2**, 210–228 [1901], especially p. 224. — A. VON ENGEL and M. STEENBECK, Elektrische Gasentladungen, Band 2, J. Springer, Berlin 1934; § 8, p. 10–12, Formulas (22), (22 a), (24 a); § 28, p. 72, Formula (125). — A. VON ENGEL, Ionized Gases, Clarendon Press, Oxford 1965, p. 224, Formula (8,5). — GORDON FRANCIS, The Glow Discharge at Low Pressure, S. FLÜGGE,

Handbuch d. Physik **22**, p. 105, Formula (35,8) [1965]. — J. D. COBINE, Gaseous Conductors, Dover Publication, New York 1958, p. 129, Formula (6,28). — T. HARA and M. UCHIDA, Jap. J. Appl. Phys. **7**, Nr. 2, p. 168, Formula (8).
⁴ W. DÄLLENBACH, Report Battelle Institute Geneva, MHD-Generator XLIII (Nov. 20, 1964).

⁵ F. BORGNIS, Z. Phys. **100**, 117–140 and 478–512 [1936].

For a series of current densities

$$j = 0.0; 0.2; 0.4; 0.6; 0.8; 1.0$$

i. e. positive charge carriers, and for initial field intensities

$$E_0 = 0.0; \pm 0.2; \pm 0.4; \pm 0.6; \pm 0.8; \pm 1.0.$$

E , n have been calculated numerically as functions of x . This yielded a complete survey of all possible solutions⁴.

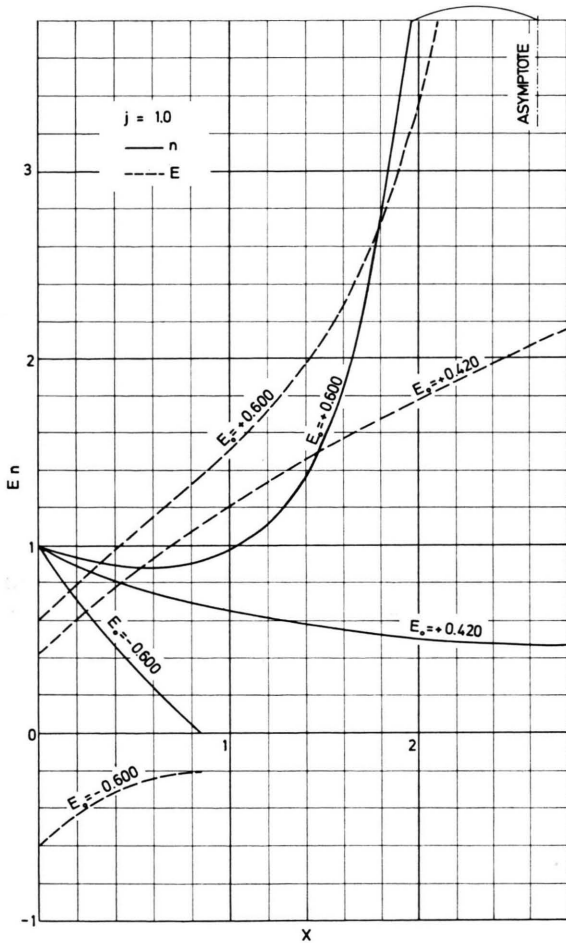


Fig. 1. Particle density n (solid lines) and electric field E (broken lines) in function of x for current density $j=1.0$ and initial values $n=1$; $E_0=0.600$; 0.420 ; -0.600 at $x=0$.

Three basically different solutions, depending on the initial field intensity E_0 , belong to each value of current density $j \geq 0$, which are illustrated by means of examples in Fig. 1, in particular for $j=1.0$.

If $E_0 = +0.600$, n , E tend to $+\infty$ with an asymptote at a finite $x=2.64$. In case $j=1.0$, this behaviour is valid for the range $E_0 > 0.420$.

If $E_0 = -0.600$, n decreases to zero at a finite $x=0.84$, whereas E attains a maximum. In case $j=1.0$, this behaviour is valid for the range $E_0 < 0.420$.

For a unipolar, one-dimensional flow according to Eqs. (6), (7), (8), there thus exists, in general, an *upper limit* x , at which either n , E tend towards infinite values (range $E_0 > 0.420$ for $j=1.0$) or n goes towards zero (range $E_0 < 0.420$ for $j=1.0$).

Only if $E_0 = 0.420$ for $j=1.0$, the flow extends up to $x = \infty$. This singular case separates the two ranges $E_0 > 0.420$ and $E_0 < 0.420$ respectively, for $j=1.0$ from each other.

The characteristics of the particle density and field intensity increase can be quite easily elucidated in the special case of *thermodynamic equilibrium*, i. e. for $j=0$. By integration of (6) one obtains the *Boltzmann-distribution*

$$n(x) = \exp \left\{ \int_0^x E(x) dx \right\} \quad (15)$$

which can be realized if, between two parallel planes at distance x , which reflect the impacting charge carriers elastically (mirror planes), the voltage

$$U = \int_0^x E(x) dx$$

is applied.

Solutions corresponding to Fig. 1 are physically meaningful only as long as the conditions for applying the phenomenological concepts mobility μ and diffusion coefficient D in Eq. (1) are fulfilled.

Necessary conditions for this are: the absolute values of field current density and diffusion current density in (1) should remain smaller than the random current density $J = \frac{1}{4} e n \bar{c}$, caused by thermal agitation, which in thermodynamic equilibrium, i. e. in the currentless state of a dense gas, passes every surface element in both directions. Thus, there must be

$$\left| \frac{e n \mu E}{\frac{1}{4} e n \bar{c}} \right| < 1 \quad \text{and} \quad \left| \frac{e D dn/dx}{\frac{1}{4} e n \bar{c}} \right| < 1.$$

\bar{c} is the arithmetic mean value of the thermal velocity of charge carriers at temperature T of the dense gas.

Because of (4) and

$$D = \frac{1}{3} \lambda \bar{c} \quad (16)$$

where λ is the mean free path between consecutive impacts of the charge carriers with the molecules of the dense neutral gas, the above inequalities become

$$\left| \frac{4}{3} \frac{e E \lambda}{k T} \right| < 1 \quad (17)$$

$$\left| \frac{4}{3} \frac{dn/dx}{n/\lambda} \right| < 1 \quad (18)$$

In terms of the units (5), i. e. in dimensionless variables, one can write instead of (17)

$$\left| \frac{4}{3} E \lambda \right| < 1 \quad (19)$$

x and λ in (18) and (19) are measured in Debye-lengths.

For thermodynamic equilibrium, i. e. for $j=0$, (18) and (19) become identical because of (6).

(17) and (19) require the work $e E \lambda$, done on a charge carrier over a distance λ in direction of E , to be smaller than $\frac{4}{3} k T$, a measure of the mean thermal energy. This condition is not sufficient. In order that the mobility μ may be a constant, independent of the field E

$$|E \lambda| \ll 1 \quad (20)$$

has to be required, i. e. the work $e E \lambda$ done by E on the charge carriers over the distance λ must be little compared to the thermal energy $\sim k T$.

If the solutions of (6), (7), (8) represented in Fig. 1 are confronted with the inequalities (18), (20) and with the previous reflections, the following ought to be retained:

a) as regards the electrode situated at $x=0$ as well as an opposite one arranged at an arbitrary x , it has to be assumed that it emits a random current density J_e which differs by $-j$ and $+j$ respectively from the random current density J_a arriving from the gas:

$$\begin{array}{ll} \text{Electrode at } x=0 & : \quad J_e = J_a + j, \\ \text{Opposite electrode at } x & : \quad J_e = J_a - j. \end{array}$$

b) If, starting from $x=0$, one approaches the singularity, where n , E tend towards infinity, (20) is no longer fulfilled from a certain x -value on.

c) If, starting from $x=0$, one approaches an x -value at which the particle density n disappears, the condition (18) is no longer met before reaching this x -value.

In case b), c), the integration should be continued from a certain cross-section on, by means of a modified basic equation substituting Eq. (1).

The rigorous solution of the set of Eqs. (1), (2) for a unipolar flow of ions in extremely pure dielectric liquids or gases is given in terms of Bessel-functions in ⁵. The discussion is more difficult than in ⁴ and the present report, where the calculating machine made a complete survey of the solutions possible. Moreover, ⁵ retains the assumption that the set of Eqs. (1), (2) is valid up to the electrode surfaces, for which boundary conditions compatible with Eq. (1) may be formulated.

III. The Solutions without Considering the Diffusion

Neglecting the diffusion, and for the time being still without the special normalization $n=1$ at $x=0$, (6), (7), (8) are replaced by

$$j = n E = n_0 E_0 = \text{const}, \quad (21)$$

$$dE/dx = n, \quad (22)$$

$$x=0; \quad n=n_0; \quad E=E_0; \quad (23)$$

n from (22) substituted in (21) and integrated gives

$$E = (E_0^2 + 2 j x)^{1/2} \quad (24)$$

$$\text{and} \quad n = j(E_0^2 + 2 j x)^{-1/2} \quad (25)$$

furthermore, the voltage over x

$$U = \int E dx = \int_0^x \frac{1}{3} j [(E_0^2 + 2 j x)^{3/2} - E_0^3] . \quad (26)$$

An expression derived from (26) for the voltage U can be found in the literature reference³. In all these cases it is assumed, contrary to the special normalization $n_0=1$ used here in the preceding paragraph II, that $E_0=0$ and consequently $n_0=\infty$. (24), (25), (26) can thus be written as follows:

$$E = (2 j x)^{1/2}, \quad (27)$$

$$n = (j/2 x)^{1/2}, \quad (28)$$

$$U = \frac{2^{3/2}}{3} j^{1/2} x^{3/2}. \quad (29)$$

From (29) follows

$$j = \frac{8}{3} U^2/x^3. \quad (30)$$

This is the formula analogous to the Child-Langmuir-space-charge formula for charge carriers falling free in vacuum, which is derived and applied in the literature reference³ mentioned above.

With the special normalization $n_0 = 1$, used in the preceding paragraph II, and $E_0 = j$, (24), (25), (26) can be written as follows:

$$E = j(1 + 2x/j)^{1/2}, \quad (31)$$

$$n = (1 + 2x/j)^{-1/2}, \quad (32)$$

$$U = \frac{1}{3} j^2 [(1 + 2x/j)^{3/2} - 1]. \quad (33)$$

In Fig. 2a and 2b E and n resp. given by (31) and (32) resp. (without considering the diffusion) are confronted with the corresponding exact solutions E , n given by (6), (7), (8) of ⁴ (with considering the diffusion).

This exemplifies the fundamental difference between the solutions with and without consideration of the diffusion.

With diffusion, there exists an upper limit of linear dimension x of the resulting flow regime for all possible current densities $j \geq 0$, whereby E and n approach infinite values.

Without diffusion, such a limit does not exist. In principle, the flow can extend up to $x = \infty$ where E tends towards infinity, but n towards zero.

Neglecting the diffusion part dn/dx of the current density in (6) is inadmissible. This is finally a consequence of the relationship between mobility and diffusion coefficient as is expressed in the Einstein-relation (4).

This situation may also be examined in a different manner. The justification to neglect the diffusion part dn/dx in (6) evidently exists only if

$$|dn/dx| \ll nE \sim j.$$

From this and from the Poisson-equation (17) follows

$$\left| \frac{dn}{dx} \right| = \left| \frac{d}{dx} (j/E) \right| = (j/E^2) n \ll j,$$

thus

$$n \ll E^2 \quad (34)$$

where n , E are dimensionless variables. The examples of solutions represented in Fig. 1 do not fulfill the inequality (34).

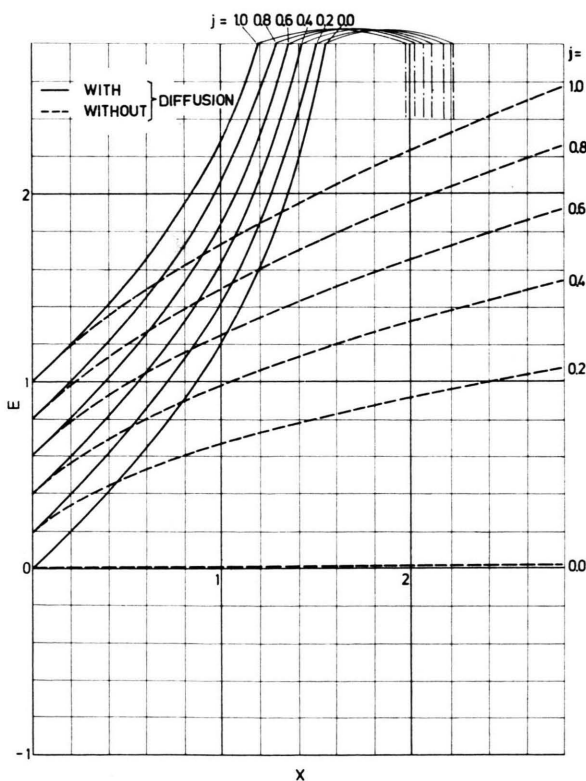


Fig. 2a. Electric field E in function of x with (solid lines) and without (broken lines) consideration of diffusion for current densities $j = 0.0; 0.2; 0.4; 0.6; 0.8; 1.0$.

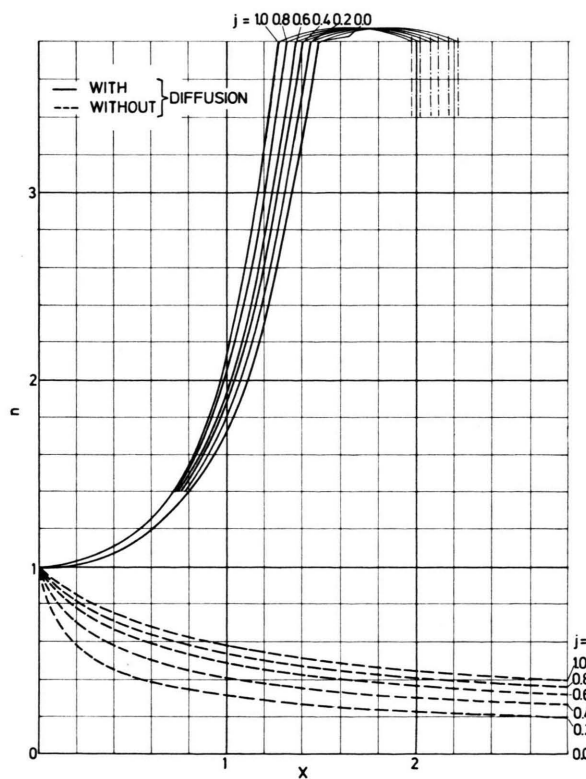


Fig. 2b. Particle density n in function of x with (solid lines) and without (broken lines) consideration of diffusion for current densities $j = 0.0; 0.2; 0.4; 0.6; 0.8; 1.0$.

With (5), (34) can be written in conventional units

$$n k T \ll \epsilon_0 E^2 \quad (\text{VA sec/cm}^3) \quad (35)$$

i. e. independent of x , in every space point, the thermal energy density $\frac{3}{2} n k T$ must be negligibly small compared to the electrostatic field energy density $\frac{1}{2} \epsilon_0 E^2$.

With $k = 1.38 \cdot 10^{-23} \quad (\text{VA sec/grad}),$
 $\epsilon_0 = 0.885 \cdot 10^{-13} \quad (\text{A sec/V cm})$
 (5) becomes

$$n T \ll 10^{10} E^2. \quad (36)$$

For a space charge formula as (30), which postu-

lates $E_0 = 0$, the charge carriers at $x = 0$ should be emitted with $T = 0$.

Acknowledgment

This investigation has been carried out at the Geneva Research Center of the Battelle Institute, and was sponsored by the Swiss Government. — The author wishes to thank the "Commission for the Encouragement of Research" at Berne, in particular Professor Dr. U. HOCHSTRASSER, Professor Dr. J. ACKERET and Professor Dr. D. RIVIER for their support and the permission to publish this paper. — Mr. A. DUVAL made the numerical calculations on an IBM 1620 and moreover contributed with great understanding to the solution of the mathematical problems.

The Propagation of Periodic Waves in a Two-level System

F. HOFELICH

Battelle Institute, Advanced Studies Center, Geneva/Switzerland

(Z. Naturforsch. **25 a**, 1452—1458 [1970] ; received 31 May 1970)

In this article the possibility for the propagation of periodic waves in a two-level system is discussed. It turns out, that in the absence of processes leading to loss and gain of energy, respectively, periodic waves only are possible. Their frequencies are always greater than the transition frequency of the two-level system. The frequency difference and the deviation of the wave profile from a sinusoidal form increases with the strength of interaction of light field and two-level systems.

After the introduction of loss and source terms it turns out that only isolated periodic waves exist. The asymptotically stable ones among them act as asymptotic solutions to all other wave solutions. The frequencies and wave forms are affected in a similar way as without loss and source terms.

1. Introduction

When by the development of the laser coherent light sources of high intensity became available, the problem of propagation of coherent light waves in a medium, absorbing near the frequency of the light wave, became of considerable interest¹⁻⁹. An excellent and exhausting survey on the state of affairs has recently been published by Arecchi and his coworkers¹⁰.

Usually, the starting point is the assumption that the absorbing medium can be replaced by a set of

two-level systems with nondegenerate energy levels. The evolution of the quantities characterizing the state of the medium, i. e. inversion and polarization, is treated quantum mechanically, whereas the propagation of the light wave and its interaction with the material system is described in a purely classical way (semi-classical theory)¹¹. One of the most important questions posed in this context is whether such a system admits wave solutions, i. e. solutions which do not change their form during propagation.

The method which is followed for the investigation of this problem consists in assuming for the

Reprints request to Dr. F. Hofelich, Battelle Institute, Geneva, Switzerland, 7, route de Drize.

¹ WITKE, J. P. and P. J. WARTER, J. Appl. Phys. **35**, 1668 [1964].

² ARECCHI, F. T. and R. BONIFACIO, IEEE J. Quant. Electr. **QE-1**, 169 [1965].

³ MIKAELIAN, A. L. and M. L. TER-MIKAELIAN, Sov. Phys. JETP **24**, 450 [1967].

⁴ MCCALL, S. L. and E. L. HAHN, Phys. Rev. Letters **18**, 908 [1967].

⁵ ARECCHI, F. T., V. DEGIORGIO and C. G. SOMEDA, Phys. Letters **27A**, 588 [1968].

⁶ ARMSTRONG, J. A. and E. COURTENS, IEEE J. Quantum Electr. **QE-4**, 411 [1968].

⁷ LAMB, G. L., Phys. Letters **28A**, 548 [1969].

⁸ MCCALL, S. L. and E. L. HAHN, Phys. Rev. **183**, 457 [1969].

⁹ EBERLY, J. H., Phys. Rev. Letters **22**, 760 [1969].

¹⁰ ARECCHI, F. T., G. L. MASSERINI, and P. SCHWENDIMANN, Rivista del Nuovo Cim. **1**, 181 [1969].

¹¹ See e. g. DAVIS, L. W., Proc. IEE **51**, 76 [1963].